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NCERT Solutions Class 9 Maths Chapter 2 - Polynomials | Comprehensive Guide

Ex 2.1 Question 1.

Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.
(i) $4x^2-3x+7$
(ii) $y^2+\sqrt{2}$
(iii) $3\sqrt{t}+t\sqrt{2}$
(iv) $y+\frac{2}{y}$
(v) $x^{10}+y^3+t^{50}$

Answer.

(i) $4x^2-3x+7$

We can observe that in the polynomial $4x^2-3x+7$, we have x as the only variable and the powers of x in each term are a whole number.

Therefore, we conclude that $4x^2-3x+7$ is a polynomial in one variable.
(ii) $y^2+\sqrt{2}$

We can observe that in the polynomial $y^2+\sqrt{2}$, we have y as the only variable and the powers of y in each term are a whole number.

Therefore, we conclude that $y^2+\sqrt{2}$ is a polynomial in one variable.
(iii) $3\sqrt{t}+t\sqrt{2}$

We can observe that in the polynomial $3\sqrt{t}+t\sqrt{2}$, we have t as the only variable and the powers of t in each term are not a whole number.

Therefore, we conclude that $3\sqrt{t}+t\sqrt{2}$ is not a polynomial in one variable.
(iv) $y+\frac{2}{y}$

We can observe that in the polynomial $y+\frac{2}{y}$, we have y as the only variable and the powers of y in each term are not a whole number.

Therefore, we conclude that $y+\frac{2}{y}$ is not a polynomial in one variable.
(v) $x^{10}+y^3+t^{30}$

We can observe that in the polynomial $x^{10}+y^3+t^{30}$, we have x, y and t as the variables and the powers of x, y and t in each term is a whole number.

Therefore, we conclude that $x^{10}+y^3+t^{30}$ is a polynomial but not a polynomial in one variable.

Ex 2.1 Question 2.

Write the coefficients of x^2 in each of the following:
(i) $2+x^2+x$
(ii) $2-x^2+x^3$
(iii) $\frac{\pi}{2}x^2+x$
(iv) $\sqrt{2}x-1$

Answer.

(i) $2+x^2+x$

The coefficient of x^2 in the polynomial $2+x^2+x$ is 1 .
(ii) $2-x^2+x^3$

The coefficient of x^2 in the polynomial $2-x^2+x^3$ is -1 .
(iii) $\frac{\pi}{2}x^2+x$

The coefficient of x^2 in the polynomial $\frac{\pi}{2}x^2+x$ is $\frac{\pi}{2}$.
(iv) $\sqrt{2}x-1$

The coefficient of x^2 in the polynomial $\sqrt{2}x-1$ is 0 .

Ex 2.1 Question 3.

Give one example each of a binomial of degree 35 , and of a monomial of degree 100 .

Answer.

The binomial of degree 35 can be $x^{35}+9$.

The binomial of degree 100 can be t^{100}.

Ex 2.1 Question 4.

Write the degree of each of the following polynomials:
(i) $p(x)=5x^3+4x^2+7x$
(ii) $p(y)=4-y^2$
(iii) $f(t)=5t-\sqrt{7}$
(iv) $f(x)=3$

Answer.

(i) $5x^3+4x^2+7x$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial $5x^3+4x^2+7x$, the highest power of the variable x is 3 .

Therefore, we conclude that the degree of the polynomial $5x^3+4x^2+7x$ is 3 .
(ii) $4-y^2$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial $4-y^2$, the highest power of the variable y is 2 .

Therefore, we conclude that the degree of the polynomial $4-y^2$ is 2 .

(iii) $5t-\sqrt{7}$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We observe that in the polynomial $5t-\sqrt{7}$, the highest power of the variable t is 1 .
Therefore, we conclude that the degree of the polynomial $5t-\sqrt{7}$ is 1 .
(iv) 3

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial 3 , the highest power of the assumed variable x is 0 .

Therefore, we conclude that the degree of the polynomial 3 is 0 .

Ex 2.1 Question 5.

Classify the following as linear, quadratic and cubic polynomials:
(i) x^2+x
(ii) $x-x^3$
(iii) $y+y^2+4$
(iv) $1+x$
(v) $3t$
(vi) r^2
(vii) $7x^3$

Answer.

(i) x^2+x

We can observe that the degree of the polynomial x^2+x is 2 .

Therefore, we can conclude that the polynomial x^2+x is a quadratic polynomial.
(ii) $x-x^3$

We can observe that the degree of the polynomial $x-x^3$ is 3 .

Therefore, we can conclude that the polynomial $x - x^3$ is a cubic polynomial.

(iii) $y + y^2 + 4$

We can observe that the degree of the polynomial $y + y^2 + 4$ is 2 .
Therefore, the polynomial $y + y^2 + 4$ is a quadratic polynomial.
(iv) $1 + x$

We can observe that the degree of the polynomial $(1 + x)$ is 1 .

Therefore, we can conclude that the polynomial $1 + x$ is a linear polynomial.
(v) $3t$

We can observe that the degree of the polynomial $(3t)$ is 1 .

Therefore, we can conclude that the polynomial $3t$ is a linear polynomial.
(vi) r^2

We can observe that the degree of the polynomial r^2 is 2 .
Therefore, we can conclude that the polynomial r^2 is a quadratic polynomial.
(vii) $7x^3$

We can observe that the degree of the polynomial $7x^3$ is 3 .

Therefore, we can conclude that the polynomial $7x^3$ is a cubic polynomial.

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Ex 2.2 Question 1.

Find the value of the polynomial $5x-4x^2+3$ at
(i) $x=0$
(ii) $x=-1$
(iii) $x=2$

Answer.

(i) Let $f(x)=5x-4x^2+3$.

We need to substitute 0 in the polynomial $f(x)=5x-4x^2+3$ to get $f(0)=5(0)-4(0)^2+3=0-0+3$
$=3$

Therefore, we conclude that at $x=0$, the value of the polynomial $5x-4x^2+3$ is 3 .
(ii) Let $f(x)=5x-4x^2+3$.

We need to substitute -1 in the polynomial $f(x)=5x-4x^2+3$ to get. $f(-1)=5(-1)-4(-1)^2+3$
$=-5-4+3$
$=-6$

Therefore, we conclude that at $x=-1$, the value of the polynomial $5x-4x^2+3$ is -6

(iii) Let $f(x)=5x-4x^2+3$.

We need to substitute 0 in the polynomial $f(x)=5x-4x^2+3$ to get $f(2)=5(2)-4(2)^2+3$
$=10-16+3$
$=-3$

Therefore, we conclude that at $x=2$, the value of the polynomial $5x-4x^2+3$ is -3 .

Ex 2.2 Question 2.

Find $p(0), p(1)$ and $p(2)$ for each of the following polynomials:
(i) $p(y)=y^2-y+1$
(ii) $p(t)=2+t+2t^2-t^3$
(iii) $p(x)=x^3$
(iv) $p(x)=(x-1)(x+1)$

Answer.

(i) $p(y)=y^2-y+1$

At $p(0)$:
$p(0)=(0)^2-0+1=1$
At $p(1)$:
$p(1)=(1)^2-1+1=1-0=1$

At $p(2)$:
$p(2)=(2)^2-2+1=4-1=3$
(ii) $p(t)=2+t+2t^2-t^3$

At$^{p(0)}$
$p(0)=2+(0)+2(0)^2-(0)^3=2$
At$^{p(1)}$
$p(1)=2+(1)+2(1)^2-(1)^3=2+1+2-1=4$
At$^{p}(2)$
$p(2)=2+(2)+2(2)^2-(2)^3=4+8-8=4$

(iii) $p(x)=(x)^3$

At $p(0)$ $p(0)=(0)^3=0$

At $p(1)$
$p(1)=(1)^3=1$

At $p(2)$
$p(2)=(2)^3=8$

(vi) $p(x)=(x-1)(x+1)$

At $p(0)$:
$p(0)=(0-1)(0+1)=(-1)(1)=-1$

At $p(1)$
$p(1)=(1-1)(2+1)=(0)(3)=0$

At $p(2)$:
$p(2)=(2-1)(2+1)=(1)(3)=3$

Ex 2.2 Question 3.

Verify whether the following are zeroes of the polynomial, indicated against them.
(i) $p(x)=3x+1, \quad x=-\frac{1}{3}$
(ii) $p(x)=5x-\pi, x=\frac{4}{5}$
(iii) $p(x)=x^2-1, \quad x=-1,1$
(iv) $p(x)=(x+1)(x-2), x=-1,2$
(v) $p(x)=x^2, x=0$
(vi) $p(x)=lx+m, x=-\frac{m}{l}$
(vii) $p(x)=3x^2-1, \quad x=-\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$
(viii) $p(x)=2x+1, \quad x=-\frac{1}{2}$

Answer.

(i) $p(x)=3x+1, \quad x=-\frac{1}{3}$

We need to check whether $p(x)=3x+1$ at $x=-\frac{1}{3}$ is equal to zero or not.

$p\left(-\frac{1}{3}\right)=3x+1=3\left(-\frac{1}{3}\right)+1=-1+1=0$

Therefore, we can conclude that $x=-\frac{1}{3}$ is a zero of the polynomial $p(x)=3x+1$.
(ii) $p(x)=5x-\pi, x=\frac{4}{5}$

We need to check whether $p(x)=5x-\pi$ at $x=\frac{4}{5}$ is equal to zero or not.
$p\left(\frac{4}{5}\right)=5\left(\frac{4}{5}\right)-\pi=4-\pi$

Therefore, $x=\frac{4}{5}$ is not a zero of the polynomial $p(x)=5x-\pi$.
(iii) $p(x)=x^2-1, x=-1,1$

We need to check whether $p(x)=x^2-1$ at $x=-1,1$ is equal to zero or not.

At $x=-1$
$p(-1)=(-1)^2-1=1-1=0$

At $x=1$

$p(1)=(1)^2-1=1-1=0$

Therefore, $x=-1,1$ are the zeros of the polynomial $p(x)=x^2-1$.
(iv) $p(x)=(x+1)(x-2), x=-1,2$

We need to check whether $p(x)=(x+1)(x-2)$ at $x=-1,2$ is equal to zero or not.

At $x=-1$
$p(-1)=(-1+1)(-1-2)=(0)(-3)=0$

At $x=2$
$p(2)=(2+1)(2-2)=(3)(0)=0$

Therefore, $x=-1,2$ are the zeros of the polynomial $p(x)=(x+1)(x-2)$.
(v) $p(x)=x^2, x=0$

We need to check whether $p(x)=x^2$ at $x=0$ is equal to zero or not.
$p(0)=(0)^2=0$

Therefore, we can conclude that $x=0$ is a zero of the polynomial $p(x)=x^2$.

(vi) $p(x)=lx+m, x=-\frac{m}{l}$

We need to check whether $p(x)=+m=0$
$p\left(-\frac{m}{l}\right)=l\left(-\frac{m}{l}\right)+m=m+m=0$

Therefore $x=-\frac{m}{l}$ is a zero of the polynomial $p(x)=lx+m$.
(vii)
$p(x)=3x^2-1, \quad x=-\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

We need to check whether
$p(x)=3x^2-1$ at $x=-\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ is equal to zero or not.
$x=\frac{-1}{\sqrt{3}}$
$p\left(-\frac{1}{\sqrt{3}}\right)=3\left(-\frac{1}{\sqrt{3}}\right)^2-1=3\left(\frac{1}{3}\right)-1=1-1=0$
$x=\frac{2}{\sqrt{3}}$
At
$$p\left(\frac{2}{\sqrt{3}}\right)=3\left(\frac{2}{\sqrt{3}}\right)^2-1=3\left(\frac{4}{3}\right)-1=4-1=3$$
Therefore, we can conclude that $x=\frac{-1}{\sqrt{3}}$ is a zero of the polynomial $p(x)=3x^2-1$ but $x=\frac{-1}{\sqrt{3}}$ is not a zero of the polynomial $p(x)=3x^2-1$.
(viii)
$p(x)=2x+1, x=-\frac{1}{2}$

We need to check whether $p(x)=2x+1$ at $x=-\frac{1}{2}$ is equal to zero or not.
$p\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)+1=-1+1=0$

Therefore, $x=-\frac{1}{2}$ is a zero of the polynomial $p(x)=2x+1$

Ex 2.2 Question 4.

Find the zero of the polynomial in each of the following cases:
(i) $p(x)=x+5$
(ii) $p(x)=x-5$
(iii) $p(x)=2x+5$
(iv) $p(x)=3x-2$
(v) $p(x)=3x$
(vi) $p(x)=ax, a\neq 0$
(vii) $p(x)=cx+d, c\neq 0, c, d$ are real numbers.

Answer.

(i) $p(x)=x+5$
$ax+b$, where $a\neq 0$ and $b\neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=x+5$ equal to 0 , we get
$x+5=0 \quad \Rightarrow x=-5$

Therefore, we conclude that the zero of the polynomial $p(x)=x+5$ is -5 .

(ii) $p(x)=x-5$
$ax+b$, where $a\neq 0$ and $b\neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=x-5$ equal to 0 , we get
$x-5=0 \quad \Rightarrow x=5$

Therefore, we conclude that the zero of the polynomial $p(x)=x-5$ is 5 .
(iii) $p(x)=2x+5$
$ax+b$, where $a\neq 0$ and $b\neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=2x+5$ equal to 0 , we get
$2x+5=0 \quad \Rightarrow x=\frac{-5}{2}$

Therefore, we conclude that the zero of the polynomial $p(x)=2x+5$ is $\frac{-5}{2}$.
(iv) $p(x)=3x-2$
$ax+b$, where $a\neq 0$ and $b\neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=3x-2$ equal to 0 , we get
$3x-2=0\Rightarrow x=\frac{2}{3}$

Therefore, we conclude that the zero of the polynomial $p(x)=3x-2$ is $\frac{2}{3}$.
(v) $p(x)=3x$
$ax+b$, where $a\neq 0$ and $b\neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=3x$ equal to 0 , we get
$3x=0 \quad \Rightarrow x=0$

Therefore, we conclude that the zero of the polynomial $p(x)=3x$ is .
(vi) $p(x)=ax, a\neq 0$
$ax+b$, where $a\neq 0$ and $b\neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=ax$ equal to 0 , we get
$ax=0\Rightarrow x=0$

Therefore, we conclude that the zero of the polynomial $p(x)=ax, a\neq 0$ is0.
(vii) $p(x)=cx+d, c\neq 0, c, d$ are real numbers.
$ax+b$, where $a\neq 0$ and $b\neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=cx+d$ equal to 0 , we get

$$cx+d=0$$
$$\Rightarrow \quad x=-\frac{d}{c}.$$

Therefore, we conclude that the zero of the polynomial $p(x)=cx+d, c\neq 0, c, d$ are real numbers. is $-\frac{d}{c}$.

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Exercise 2.3 Question 1.

Determine which of the following polynomials has $(x+1)$ a factor:
(i) x^3+x^2+x+1
(ii) $x^4+x^3+x^2+x+1$
(iii) $x^4+3x^3+3x^2+x+1$
(iv) $x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$

Answer.

(i) x^3+x^2+x+1

While applying the factor theorem, we get
$p(x)=x^3+x^2+x+1$
$p(-1)=(-1)^3+(-1)^2+(-1)+1$
$=-1+1-1+1$
$=0$

We conclude that on dividing the polynomial x^3+x^2+x+1 by $(x+1)$, we get the remainder as0.

Therefore, we conclude that $(x+1)$ is a factor of x^3+x^2+x+1.
(ii) $x^4+x^3+x^2+x+1$

While applying the factor theorem, we get
$p(x)=x^4+x^3+x^2+x+1$
$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$
$=1-1+1-1+1$
$=1$

We conclude that on dividing the polynomial $x^4+x^3+x^2+x+1$ by $(x+1)$, we will get the remainder as1, which is not 0 .

Therefore, we conclude that $(x+1)$ is not a factor of $x^4+x^3+x^2+x+1$.
(iii) $x^4+3x^3+3x^2+x+1$

While applying the factor theorem, we get

$p(x)=x^4+3x^3+3x^2+x+1$
$p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$
$=1-3+3-1+1$
$=1$

We conclude that on dividing the polynomial $x^4+3x^3+3x^2+x+1$ by $(x+1)$, we will get the remainder as 1 , which is not 0 .

Therefore, we conclude that $(x+1)$ is not a factor of $x^4+3x^3+3x^2+x+1$.
(iv) $x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$

While applying the factor theorem, we get

$p(x)=x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$

$p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+\sqrt{2}$

$=-1-1+2+\sqrt{2}+\sqrt{2}$

$=2\sqrt{2}$

We conclude that on dividing the polynomial $x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$ by $(x+1)$, we will get the remainder as $2\sqrt{2}$, which is not 0 .

Therefore, we conclude that $(x+1)$ is not a factor of $x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$.

Exercise 2.3 Question 2.

Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x)=2x^3+x^2-2x-1, g(x)=x+1$

(ii) $p(x)=x^3+3x^2+3x+1, g(x)=x+2$

(iii) $p(x)=x^3-4x^2+x+6, g(x)=x-3$

Answer.

(i) $p(x)=2x^3+x^2-2x-1, g(x)=x+1$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a)=0$.

We can conclude that $g(x)$ is a factor of $p(x)$, if $p(-1)=0$.

$p(-1)=2(-1)^3+(-1)^2-2(-1)-1$

$=2+1-1-2$

$=0$

Therefore, we conclude that the $g(x)$ is a factor of $p(x)$.

(ii) $p(x)=x^3+3x^2+3x+1, g(x)=x+2$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a)=0$.

We can conclude that $g(x)$ is a factor of $p(x)$, if $p(-2)=0$.

$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$

$=-8+12-6+1$

$=-1$

Therefore, we conclude that the $g(x)$ is not a factor of $p(x)$.

(iii) $p(x)=x^3-4x^2+x+6, g(x)=x-3$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a)=0$.

We can conclude that $g(x)$ is a factor of $p(x)$, if $p(3)=0$.

$p(3)=(3)^3-4(3)^2+(3)+6$

$=27-36+3+6$

$=0$

Therefore, we conclude that the $g(x)$ is a factor of $p(x)$.

Exercise 2.3 Question 3.

Find the value of k, if $x-1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x)=x^2+x+k$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

(iv) $p(x)=kx^2-3x+k$

Answer.

(i) $p(x)=x^2+x+k$

We know that according to the factor theorem

$p(a)=0$, if $x-a$ is a factor of $p(x)$

We conclude that if $(x-1)$ is a factor of $p(x)=x^2+x+k$, then $p(1)=0$.

$p(1)=(1)^2+(1)+k=0, \text{ or}$

$k+2=0$

$k=-2$

Therefore, we can conclude that the value of k is -2 .

(ii) $p(x)=2x^2+kx+\sqrt{2}$

We know that according to the factor theorem

$p(a)=0$, if $x-a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x)=2x^2+kx+\sqrt{2}$, then $p(1)=0$.
$p(1)=2(1)^2+k(1)+\sqrt{2}=0$, or
$2+k+\sqrt{2}=0$
$k=-(2+\sqrt{2})$.

Therefore, we can conclude that the value of k is $-(2+\sqrt{2})$.
(iii) $p(x)=kx^2-\sqrt{2}x+1$

We know that according to the factor theorem

$p(a)=0$, if $x-a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x)=kx^2-\sqrt{2}x+1$, then $p(1)=0$.
$p(1)=k(1)^2-\sqrt{2}(1)+1=0$ or
$k-\sqrt{2}+1=0$
$k=\sqrt{2}-1$

Therefore, we can conclude that the value of k is $\sqrt{2}-1$.
(iv) $p(x)=kx^2-3x+k$

We know that according to the factor theorem
$p(a)=0$, if $x-a$ is a factor of p(x)

We conclude that if $(x-1)$ is a factor of $p(x)=kx^2-3x+k$, then $p(1)=0$.
$p(1)=k(1)^2-3(1)+k$; or $2k-3=0 \quad \Rightarrow k=\frac{3}{2}$

Therefore, we can conclude that the value of k is $\frac{3}{2}$.

Exercise 2.3 Question 4.

Factorize:
(i) $12x^2-7x+1$
(ii) $2x^2+7x+3$
(iii) $6x^2+5x-6$
(iv) $3x^2-x-4$

Answer.

(i) $12x^2-7x+1$
$12x^2-7x+1=12x^2-3x-4x+1$
$=3x(4x-1)-1(4x-1)$
$=(3x-1)(4x-1)$.

Therefore, we conclude that on factorizing the polynomial $12x^2-7x+1$, we get $(3x-1)(4x-1)$.
(ii) $2x^2+7x+3$
$2x^2+7x+3=2x^2+6x+x+3$
$=2x(x+3)+1(x+3)$
$=(2x+1)(x+3)$.

Therefore, we conclude that on factorizing the polynomial $2x^2+7x+3$, we get $(2x+1)(x+3)$.
(iii) $6x^2+5x-6$
$6x^2+5x-6=6x^2+9x-4x-6$
$=3x(2x+3)-2(2x+3)$
$=(3x-2)(2x+3)$.

Therefore, we conclude that on factorizing the polynomial $6x^2+5x-6$, we get $(3x-2)(2x+3)$.
(iv) $3x^2-x-4$
$3x^2-x-4=3x^2+3x-4x-4$
$=3x(x+1)-4(x+1)$
$=(3x-4)(x+1)$

Therefore, we conclude that on factorizing the polynomial $3x^2-x-4$, we get $(3x-4)(x+1)$.

Exercise 2.3 Question 5.

Factorize:
(i) x^3-2x^2-x+2
(ii) x^3-3x^2-9x-5
(iii) $x^3+13x^2+32x+20$
(iv) $2y^3+y^2-2y-1$

Answer.

(i) x^3-2x^2-x+2

We need to consider the factors of 2 , which are $\pm 1, \pm 2$.

Let us substitute 1 in the polynomial x^3-2x^2-x+2, to get
$(1)^3-2(1)^2-(1)+2=1-1-2+2=0$

Thus, according to factor theorem, we can conclude that $(x-1)$ is a factor of the polynomial x^3-2x^2-x+2.

Let us divide the polynomial x^3-2x^2-x+2 by $(x-1)$, to get

$$
\begin{array}{l}
\qquad\qquad x^2-x-2 \\
x-1\overline{\big)\,x^3-2x^2-x+2} \\
\qquad\; \underline{x^3-x^2} \\
\qquad\;\; {}_{-}\quad {}_{+} \\
\qquad\; -x^2-x \\
\qquad\; \underline{-x^2+x} \\
\qquad\;\; {}_{+}\quad {}_{-} \\
\qquad\; -2x+2 \\
\qquad\; \underline{-2x+2} \\
\qquad\;\; {}_{+}\quad {}_{-} \\
\qquad\qquad \underline{0}
\end{array}
$$

$x^3-2x^2-x+2=(x-1)\left(x^2-x-2\right)$.
$x^3-2x^2-x+2=(x-1)\left(x^2-x-2\right)$.
$=(x-1)\left(x^2+x-2x-2\right)$
$=(x-1)[x(x+1)-2(x+1)]$
$=(x-1)(x-2)(x+1)$.

Therefore, we can conclude that on factorizing the polynomial x^3-2x^2-x+2, we get $(x-1)(x-2)(x+1)$.
(ii) x^3-3x^2-9x-5

We need to consider the factors of -5 , which are $\pm 1, \pm 5$.

Let us substitute 1 in the polynomial x^3-3x^2-9x-5, to get
$(-1)^3-3(-1)^2-9(-1)-5=-1-3+9-5=0$

Thus, according to factor theorem, we can conclude that $^{(x+1)}$ is a factor of the polynomial x^3-3x^2-9x-5.

Let us divide the polynomial x^3-3x^2-9x-5 by $(x+1)$, to get

$$
\begin{array}{l}
\qquad\qquad x^2-4x-5 \\
x+1\overline{\big)\,x^3-3x^2-9x-5} \\
\qquad\; \underline{x^3+x^2} \\
\qquad\;\; {}_{-}\quad {}_{-} \\
\qquad\; -4x^2-9x \\
\qquad\; \underline{-4x^2-4x} \\
\qquad\;\; {}_{+}\quad {}_{+} \\
\qquad\; -5x-5 \\
\qquad\; \underline{-5x-5} \\
\qquad\;\; {}_{+}\quad {}_{+} \\
\qquad\qquad \underline{0}
\end{array}
$$

$x^3-3x^2-9x-5=(x+1)\left(x^2-4x-5\right)$
$=(x+1)\left(x^2+x-5x-5\right)$
$=(x+1)[x(x+1)-5(x+1)]$
$=(x+1)(x-5)(x+1)$.

Therefore, we can conclude that on factorizing the polynomial x^3-3x^2-9x-5, we get $(x+1)(x-5)(x+1)$
(iii) $x^3+13x^2+32x+20$

We need to consider the factors of 20 , which are $\pm 5, \pm 4, \pm 2, \pm 1$.

Let us substitute -1 in the polynomial $x^3+13x^2+32x+20$, to get
$(-1)^3+13(-1)^2+32(-1)+20=-1+13-32+20=-20+20=0$

$$
\begin{array}{l}
\qquad\qquad x^2+12x+20 \\
x+1\overline{\big)\,x^3+13x^2+32x+20} \\
\qquad\; \underline{x^3+x^2} \\
\qquad\;\; {}_{-}\quad {}_{-} \\
\qquad\; 12x^2+32x \\
\qquad\; \underline{12x^2+12x} \\
\qquad\;\; {}_{-}\quad {}_{-} \\
\qquad\; 20x+20 \\
\qquad\; \underline{20x+20} \\
\qquad\;\; {}_{-}\quad {}_{-} \\
\qquad\qquad \underline{0}
\end{array}
$$

$x^3+13x^2+32x+20=(x+1)\left(x^2+12x+20\right)$
$=(x+1)\left(x^2+2x+10x+20\right)$
$=(x+1)[x(x+2)+10(x+2)]$
$=(x+1)(x+10)(x+2).$

Therefore, we can conclude that on factorizing the polynomial $x^3+13x^2+32x+20$, we get $(x+1)(x-10)(x+2)$
(iv) $2y^3+y^2-2y-1$

We need to consider the factors of -1 , which are ± 1.

Let us substitute 1 in the polynomial $2y^3+y^2-2y-1$, to get
$2(1)^3+(1)^2-2(1)-1=2+1-2-1=3-3=0$

Thus, according to factor theorem, we can conclude that $(y-1)$ is a factor of the polynomial $2y^3+y^2-2y-1$.

Let us divide the polynomial $2y^3+y^2-2y-1$ by $(y-1)$, to get

$$\begin{array}{r|l} & 2y^2+3y+1 \\ y-1 & 2y^3+y^2-2y-1 \\ & \underline{2y^3-2y^2} \\ & \quad - \quad + \\ & 3y^2-2y \\ & \underline{3y^2-3y} \\ & \quad - \quad + \\ & y-1 \\ & \underline{y-1} \\ & - \quad - \\ & \quad 0 \end{array}$$

$2y^3+y^2-2y-1=(y-1)\left(2y^2+3y+1\right)$
$=(y-1)\left(2y^2+2y+y+1\right)$
$=(y-1)[2y(y+1)+1(y+1)]$
$=(y-1)(2y+1)(y+1).$

Therefore, we can conclude that on factorizing the polynomial $2y^3+y^2-2y-1$, we get $(y-1)(2y+1)(y+1)$.

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Ex 2.4 Question 1.

Use suitable identities to find the following products:
(i) $(x+4)(x+10)$
(ii) $(x+8)(x-10)$
(iii) $(3x+4)(3x-5)$
(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$
(v) $(3-2x)(3+2x)$

Answer.

(i) $(x+4)(x+10)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab.$

We need to apply the above identity to find the product $(x+4)(x+10)$
$(x+4)(x+10)=x^2+(4+10)x+(4\times 10)$
$=x^2+14x+40$

Therefore, we conclude that the product $(x+4)(x+10)$ is $x^2+14x+40$.

(ii) $(x+8)(x-10)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab.$

We need to apply the above identity to find the product $(x+8)(x-10)$
$(x+8)(x-10)=x^2+[8+(-10)]x+[8\times(-10)]$
$=x^2-2x-80.$

Therefore, we conclude that the product $(x+8)(x-10)$ is $x^2-2x-80$.
(iii) $(3x+4)(3x-5)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab.$

We need to apply the above identity to find the product $(3x+4)(3x-5)$
$(3x+4)(3x-5)=(3x)^2+[4+(-5)]3x+[4\times(-5)]$
$=9x^2-3x-20.$

Therefore, we conclude that the product $(3x+4)(3x-5)$ is $9x^2-3x-20$.

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

We know that $(x+y)(x-y)=x^2-y^2$.

We need to apply the above identity to find the product $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$=\left(y^2\right)^2-\left(\frac{3}{2}\right)^2=y^4-\frac{9}{4}$.

Therefore, we conclude that the product $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$ is $\left(y^4-\frac{9}{4}\right)$.

(v) $(3+2x)(3-2x)$

We know that $(x+y)(x-y)=x^2-y^2$.

We need to apply the above identity to find the product $(3+2x)(3-2x)$

$(3+2x)(3-2x)=(3)^2-(2x)^2$

$=9-4x^2$.

Therefore, we conclude that the product $(3+2x)(3-2x)$ is $\left(9-4x^2\right)$.

Ex 2.4 Question 2.

Evaluate the following products without multiplying directly:

(i) 103×107

(ii) 98×96

(iii) 104×96

Answer.

(i) 103×107

103×107 can al so be written as $(100+3)(100+7)$.

We can observe that, we can apply the identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$(100+3)(100+7)=(100)^2+(3+7)(100)+3\times 7$

$=10000+1000+21$

$=11021$

Therefore, we conclude that the value of the product 103×107 is 11021 .

(ii) 95×96

95×96 can also be written as $(100-5)(100-4)$

We can observe that, we can apply the identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$(100-5)(100-4)=(100)^2+[(-5)+(-4)](100)+(-5)\times(-4)$

$=10000-900+20$

$=9120$

Therefore, we conclude that the value of the product 95×96 is 9120 .

(iii) 104×96

104×96 can also be written as $(100+4)(100-4)$.

We can observe that, we can apply the identity $(x+y)(x-y)=x^2-y^2$ with respect to the expression $(100+4)(100-4)$, to get

$(100+4)(100-4)=(100)^2-(4)^2$

$=10000-16$

$=9984$

Therefore, we conclude that the value of the product 104×96 is 9984 .

Ex 2.4 Question 3.

Factorize the following using appropriate identities:

(i) $9x^2+6xy+y^2$

(ii) $4y^2-4y+1$

(iii) $x^2-\frac{y^2}{100}$

Answer.

(i) $9x^2+6xy+y^2$

$9x^2+6xy+y^2=(3x)^2+2\times 3x\times y+(y)^2$

We can observe that, we can apply the identity $(x+y)^2=x^2+2xy+y^2$

$\Rightarrow(3x)^2+2\times 3x\times y+(y)^2=(3x+y)^2$.

(ii) $4y^2-4y+1$

$4y^2-4y+1=(2y)^2-2\times 2y\times 1+(1)^2$

We can observe that, we can apply the identity $(x-y)^2=x^2-2xy+y^2$

$\Rightarrow(2y)^2-2\times 2y\times 1+(1)^2=(2y-1)^2$.

(iii) $x^2-\frac{y^2}{100}$

We can observe that, we can apply the identity $(x)^2-(y)^2=(x+y)(x-y)$

$\Rightarrow (x)^2-\left(\frac{y}{10}\right)^2=\left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$

Ex 2.4 Question 4.

Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$

(ii) $(2x-y+z)^2$

(iii) $(-2x+3y+2z)^2$

(iv) $(3a-7b-c)^2$

(v) $(-2x+5y-3z)^2$

(vi) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

Answer.

(i) $(x+2y+4z)^2$

We know that $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(x+2y+4z)^2$.

$$\begin{aligned}(x+2y+4z)^2&=(x)^2+(2y)^2+(4z)^2+2\times x\times 2y+2\times 2y\times 4z+2\times 4z\times x\\&=x^2+4y^2+16z^2+4xy+16yz+8zx\end{aligned}$$

(ii) $(2x-y+z)^2$

We know that $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(2x-y+z)^2$.

$$\begin{aligned}(2x-y+z)^2&=[2x+(-y)+z]^2\\&=(2x)^2+(-y)^2+(z)^2+2\times 2x\times(-y)+2\times(-y)\times z+2\times z\times 2x\\&=4x^2+y^2+z^2-4xy-2yz+4zx\end{aligned}$$

(iii) $(-2x+3y+2z)^2$

We know that $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(-2x+3y+2z)^2$.

$$\begin{aligned}(-2x+3y+2z)^2&=[(-2x)+3y+2z]^2\\&=(-2x)^2+(3y)^2+(2z)^2+2\times(-2x)\times 3y+2\times 3y\times 2z+2\times 2z\times(-2x)\\&=4x^2+9y^2+4z^2-12xy+12yz-8zx\end{aligned}$$

(iv) $(3a-7b-c)^2$

We know that $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(3a-7b-c)^2$.

$$\begin{aligned}(3a-7b-c)^2&=[3a+(-7b)+(-c)]^2\\&=(3a)^2+(-7b)^2+(-c)^2+2\times 3a\times(-7b)+2\times(-7b)\times(-c)+2\times(-c)\times 3a\\&=9a^2+49b^2+c^2-42ab+14bc-6ac\end{aligned}$$

(v) $(-2x+5y-3z)^2$

We know that $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(-2x+5y-3z)^2$.

$$\begin{aligned}&(-2x+5y-3z)^2=[(-2x)+5y+(-3z)]^2\\&=(-2x)^2+(5y)^2+(-3z)^2+2\times(-2x)\times 5y+2\times 5y\times(-3z)+2\times(-3z)\times(-2x)\\&=4x^2+25y^2+9z^2-20xy-30yz+12zx.\end{aligned}$$

(vi) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

We know that $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$.

$$\begin{aligned}&\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2=\left[\frac{a}{4}+\left(-\frac{b}{2}\right)+1\right]^2\\&=\left(\frac{a}{4}\right)^2+\left(-\frac{b}{2}\right)^2+(1)^2+2\times\frac{a}{4}\times\left(-\frac{b}{2}\right)+2\times\left(-\frac{b}{2}\right)\times 1+2\times 1\times\frac{a}{4}\\&=\frac{a^2}{16}+\frac{b^2}{4}+1-\frac{ab}{4}-b+\frac{a}{2}.\end{aligned}$$

Ex 2.4 Question 5.

Factorize:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Answer.

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

The expression $4x^2+9y^2+16z^2+12xy-24yz-16xz$ can also be written as

$(2x)^2+(3y)^2+(-4z)^2+2\times 2x\times 3y+2\times 3y\times(-4z)+2\times(-4z)\times 2x$

We can observe that, we can apply the identity $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$ with respect to the expression $(2x)^2+(3y)^2+(-4z)^2+2\times 2x\times 3y+2\times 3y\times(-4z)+2\times(-4z)\times 2x$, to get
$(2x+3y-4z)^2$

Therefore, we conclude that after factorizing the expression $4x^2+9y^2+16z^2+12xy-24yz-16xz$, we get $(2x+3y-4z)^2$.
(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

We need to factorize the expression $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$.

The expression $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$ can also be written as

$(-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+2\times(-\sqrt{2}x)\times y+2\times y\times(2\sqrt{2}z)+2\times(2\sqrt{2}z)\times(-\sqrt{2}x)$.$

We can observe that, we can apply the identity $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$ with respect to the expression $(-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+2\times(-\sqrt{2}x)\times y+2\times y\times(2\sqrt{2}z)+2\times(2\sqrt{2}z)\times(-\sqrt{2}x)$, to get $(-\sqrt{2}x+y+2\sqrt{2}z)^2$

Therefore, we conclude that after factorizing the expression $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$, we get $(-\sqrt{2}x+y+2\sqrt{2}z)^2$.

Ex 2.4 Question 6.

Write the following cubes in expanded form:
(i) $(2x+1)^3$
(ii) $(2a-3b)^3$
(iii) $\left(\frac{3}{2}x+1\right)^3$
(iv) $\left(x-\frac{2}{3}y\right)^3$

Answer.

(i) $(2x+1)^3$

We know that $(x+y)^3=x^3+y^3+3xy(x+y)$.
$$\begin{aligned}\therefore (2x+1)^3 &= (2x)^3+(1)^3+3\times 2x\times 1(2x+1)\\ &= 8x^3+1+6x(2x+1)\\ &= 8x^3+12x^2+6x+1.\end{aligned}$$

Therefore, the expansion of the expression $(2x+1)^3$ is $8x^3+12x^2+6x+1$.
(ii) $(2a-3b)^3$

We know that $(x-y)^3=x^3-y^3-3xy(x-y)$.

$$\begin{aligned}\therefore (2a-3b)^3 &= (2a)^3-(3b)^3-3\times 2a\times 3b(2a-3b)\\ &= 8a^3-27b^3-18ab(2a-3b)\\ &= 8a^3-36a^2b+54ab^2-27b^3.\end{aligned}$$

Therefore, the expansion of the expression $(2a-3b)^3$ is $8a^3-36a^2b+54ab^2-27b^3$.
(iii) $\left(\frac{3}{2}x+1\right)^3$

We know that $(x+y)^3=x^3+y^3+3xy(x+y)$.
$$\begin{aligned}\left(\frac{3}{2}x+1\right)^3 &= \left(\frac{3}{2}x\right)^3+(1)^3+3\times\frac{3}{2}x\times 1\left(\frac{3}{2}x+1\right)\therefore\\ &= \frac{27}{8}x^3+1+\frac{9}{2}x\left(\frac{3}{2}x+1\right)\\ &= \frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1\end{aligned}$$

Therefore, the expansion of the expression $\left(\frac{3}{2}x+1\right)^3$ is $\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$.

(iv) $\left(x-\frac{2}{3}y\right)^3$

We know that $

$$\begin{aligned}&\therefore \left(x-\frac{2}{3}y\right)^3=(x)^3-\left(\frac{2}{3}y\right)^3-3\times x\times\frac{2}{3}y\left(x-\frac{2}{3}y\right)\\ &= x^3-\frac{8}{27}y^3-2xy\left(x-\frac{2}{3}y\right)\\ &= x^3-2x^2y+\frac{4}{3}xy^2-\frac{8}{27}y^3.\end{aligned}$$

Therefore, the expansion of the expression $\left(x-\frac{2}{3}y\right)^3_{\text{is}}$ $x^3-2x^2y+\frac{4}{3}xy^2-\frac{8}{27}y^3$.

x-y)^3=x^3-y^3-3 x y(x-y)$.

Ex 2.4 Question 7.

Evaluate the following using suitable identities:
(i) $(99)^3$
(ii) $(102)^3$
(iii) $(998)^3$

Answer.

(i) $(99)^3$
$(99)^3$ can al so be written as $(100-1)^3$.

Using identity, $(x-y)^3=x^3-y^3-3xy(x-y)$
$(100-1)^3=(100)^3-(1)^3-3\times 100\times 1(100-1)$
$=1000000-1-300(99)$
$=999999-29700$
$=970299$
(ii) $(102)^3$
$(102)^3$ can also be written as $(100+2)^3$.

Using identity $(x+y)^3=x^3+y^3+3xy(x+y)$
$(100+2)^3=(100)^3+(2)^3+3\times 100\times 2(100+2)$
$=1000000+8+600(102)$
$=1000008+61200$
$=1061208$
(iii) $(998)^3$
$(998)^3$ can also be written as $(1000-2)^3$.

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$
$(1000-2)^3=(1000)^3-(2)^3-3\times 1000\times 2(1000-2)$
$=1000000000-8-6000(998)$
$=999999992-5988000$
$=994011992$

Ex 2.4 Question 8.

Factorize each of the following:
(i) $8a^3+b^3+12a^2b+6ab^2$ (ii) $8a^3-b^3-12a^2b+6ab^2$
(iii) $27-125a^3-135a+225a^2$ (iv) $64a^3-27b^3-144a^2b+108ab^2$
(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

Answer.

(i) $8a^3+b^3+12a^2b+6ab^2$

The expression $8a^3+b^3+12a^2b+6ab^2$ can also be written as
$=(2a)^3+(b)^3+3\times 2a\times 2a\times b+3\times 2a\times b\times b$
$=(2a)^3+(b)^3+3\times 2a\times b(2a+b)$.

Using identity $(x+y)^3=x^3+y^3+3xy(x+y)$ withrespect to the expression $(2a)^3+(b)^3+3\times 2a\times b(2a+b)$, we get $(2a+b)^3$.

Therefore, after factorizing the expression $8a^3+b^3+12a^2b+6ab^2$, we get $(2a+b)^3$.
(ii) $8a^3-b^3-12a^2b+6ab^2$

The expression $8a^3-b^3-12a^2b+6ab^2$ can also be written as
$=(2a)^3-(b)^3-3\times 2a\times 2a\times b+3\times 2a\times b\times b$
$=(2a)^3-(b)^3-3\times 2a\times b(2a-b)$.

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$ with respect to the expression $(2a)^3-(b)^3-3\times 2a\times b(2a-b)$ we get $(2a-b)^3$.

Therefore, after factorizing the expression $8a^3-b^3-12a^2b+6ab^2$, we get $(2a-b)^3$.
(iii) $27-125a^3-135a+225a^2$

The expression $27-125a^3-135a+225a^2$ can also be written as
$=(3)^3-(5a)^3-3\times 3\times 3\times 5a+3\times 3\times 5a\times 5a$
$=(3)^3-(5a)^3+3\times 3\times 5a(3-5a)$.

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$ with respect to the expression $(3)^3-(5a)^3+3\times 3\times 5a(3-5a)$, we get $(3-5a)^3$.

Therefore, after factorizing the expression $27-125a^3-135a+225a^2$, we get $(3-5a)^3$.
(iv) $64a^3-27b^3-144a^2b+108ab^2$

The expression $64a^3-27b^3-144a^2b+108ab^2$ can also be written as
$=(4a)^3-(3b)^3-3\times 4a\times 4a\times 3b+3\times 4a\times 3b\times 3b$
$=(4a)^3-(3b)^3-3\times 4a\times 3b(4a-3b)$.

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$ with respect to the expression $(4a)^3-(3b)^3-3\times 4a\times 3b(4a-3b)$, we get $(4a-3b)^3$.

Therefore, after factorizing the expression $64a^3-27b^3-144a^2b+108ab^2$, we get $(4a-3b)^3$.
(v)
$27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

The expression $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$ can also be written as

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times 3p \times \frac{1}{6} + 3 \times 3p \times \frac{1}{6} \times \frac{1}{6}$$

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times \frac{1}{6}\left(3p - \frac{1}{6}\right)$$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ with respect to the expression

$(3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times \frac{1}{6}\left(3p - \frac{1}{6}\right)$

to get $\left(3p - \frac{1}{6}\right)^3$.

Therefore, after factorizing the expression $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$, we get $\left(3p - \frac{1}{6}\right)^3$.

Ex 2.4 Question 9.

Verify:

(i) $x^3 + y^3 = (x+y)\left(x^2 - xy + y^2\right)$

(ii) $x^3 - y^3 = (x-y)\left(x^2 + xy + y^2\right)$

Answer.

(i) $x^3 + y^3 = (x+y)\left(x^2 - xy + y^2\right)$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$\Rightarrow x^3 + y^3 = (x+y)^3 - 3xy(x+y)$

$= (x+y)\left[(x+y)^2 - 3xy\right]$

$\because$ We know that $(x+y)^2 = x^2 + 2xy + y^2$

$\therefore x^3 + y^3 = (x+y)\left(x^2 + 2xy + y^2 - 3xy\right)$

$= (x+y)\left(x^2 - xy + y^2\right)$

Therefore, the desired result has been verified.

(ii) $x^3 - y^3 = (x-y)\left(x^2 + xy + y^2\right)$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$

$= (x-y)\left[(x-y)^2 + 3xy\right]$

$\because$ We know that $(x-y)^2 = x^2 - 2xy + y^2$

$\therefore x^3 - y^3 = (x-y)\left(x^2 - 2xy + y^2 + 3xy\right)$

$= (x-y)\left(x^2 + xy + y^2\right)$

Therefore, the desired result has been verified.

Ex 2.4 Question 10.

Factorize:

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

Answer.

(i) $27y^3 + 125z^3$

The expression $27y^3 + 125z^3$ can also be written as $(3y)^3 + (5z)^3$.

We know that $x^3 + y^3 = (x+y)\left(x^2 - xy + y^2\right)$.

$(3y)^3 + (5z)^3 = (3y + 5z)\left[(3y)^2 - 3y \times 5z + (5z)^2\right]$

$= (3y + 5z)\left(9y^2 - 15yz + 25z^2\right)$.

(ii) $64m^3 - 343n^3$

The expression $64m^3 - 343n^3$ can also be written as $(4m)^3 - (7n)^3$.

We know that $x^3 - y^3 = (x-y)\left(x^2 + xy + y^2\right)$.

$(4m)^3 - (7n)^3 = (4m - 7n)\left[(4m)^2 + 4m \times 7n + (7n)^2\right]$

$= (4m - 7n)\left(16m^2 + 28mn + 49n^2\right)$

Therefore, we conclude that after factorizing the expression $64m^3 - 343n^3$, we get $(4m - 7n)\left(16m^2 + 28mn + 49n^2\right)$.

Ex 2.4 Question 11.

Factorize: $27x^3 + y^3 + z^3 - 9xyz$

Answer.

The expression $27x^3 + y^3 + z^3 - 9xyz$ can also be written as

$(3x)^3 + (y)^3 + (z)^3 - 3 \times 3x \times y \times z$.

We know that $x^3+y^3+z^3-3xyz=(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)$.

$$\therefore (3x)^3+(y)^3+(z)^3-3\times 3x\times y\times z=(3x+y+z)\left[(3x)^2+(y)^2+(z)^2-3x\times y-y\times z-z\times 3x\right]$$
$$=(3x+y+z)\left(9x^2+y^2+z^2-3xy-yz-3xz\right).$$

Therefore, we conclude that after factorizing the expression $27x^3+y^3+z^3-9xyz$, we get $(3x+y+z)\left(9x^2+y^2+z^2-3xy-yz-3xz\right)$.

Ex 2.4 Question 12.

Verify that
$x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$

Answer.
LHS is $x^3+y^3+z^3-3xyz$ and RHS is $\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$.

We know that $x^3+y^3+z^3-3xyz=(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)$.

And also, we know that $(x-y)^2=x^2-2xy+y^2$.

$$\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$$
$$\frac{1}{2}(x+y+z)\left[\left(x^2-2xy+y^2\right)+\left(y^2-2yz+z^2\right)+\left(z^2-2xz+x^2\right)\right]$$
$$\frac{1}{2}(x+y+z)\left(2x^2+2y^2+2z^2-2xy-2yz-2zx\right)$$
$$(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right).$$

Therefore, we can conclude that the desired result is verified.

Ex 2.4 Question 13.

If $x+y+z=0$, show that $x^3+y^3+z^3=0$.

Answer.

We know that $x^3+y^3+z^3-3xyz=(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)$.

We need to substitute $x^3+y^3+z^3=0$ in $x^3+y^3+z^3-3xyz=(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)$, to get

$$x^3+y^3+z^3-3xy=(0)=\left(x^2+y^2+z^2-xy-yz-zx\right),$$
$$x^3+y^3+z^3-3xyz=0$$
$$\Rightarrow x^3+y^3+z^3=3xyz.$$

Therefore, the desired result is verified.

Ex 2.4 Question 14.

Without actually calculating the cubes, find the value of each of the following:
(i) $(-12)^3+(7)^3+(5)^3$
(ii) $(28)^3+(-15)^3+(-13)^3$

Answer.

(i) $(-12)^3+(7)^3+(5)^3$

Let $a=-12, b=7$ and $c=5$

We know that, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Here, $a+b+c=-12+7+5=0$

$$\therefore (-12)^3+(7)^3+(5)^3=3(-12)(7)(5)$$
$$=-1260$$

(ii) $(28)^3+(-15)^3+(-13)^3$

Let $a=28, b=-15$ and $c=-13$

We know that, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Here, $a+b+c=28-15-13=0$

$$\therefore (28)^3+(-15)^3+(-13)^3=3(28)(-15)(-13)$$
$$=16380$$

Ex 2.4 Question 15.

Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:
(i) Are: $25a^2-35a+12$
(ii) Are: $35y^2+13y-12$

Answer.

(i) Area : $25a^2-35a+12$

The expression $25a^2-35a+12$ can also be written as $25a^2-15a-20a+12$.
$25a^2-15a-20a+12=5a(5a-3)-4(5a-3)$
$=(5a-4)(5a-3)$.

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $25a^2-35a+12$ is Length $=(5a-4)$ and Breadth $=(5a-3)$.
(ii) Area : $35y^2+13y-12$

The expression $35y^2+13y-12$ can also be written as $35y^2+28y-15y-12$.
$35y^2+28y-15y-12=7y(5y+4)-3(5y+4)$
$=(7y-3)(5y+4)$.

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $35y^2+13y-12$ is Length $=(7y-3)$ and Breadth $=(5y+4)$.

Ex 2.4 Question 16.

What are the possible expressions for the dimensions of the cuboids whose volumes are given below?
(i) Volume: $3x^2-12x$
(ii) Volume: $12ky^2+8ky-20k$

Answer.

(i) Volume : $3x^2-12x$

The expression $3x^2-12x$ can also be written as $3\times x\times(x-4)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $3x^2-12x_{\text{is}}\,3, x$ and $(x-4)$.
(ii) Volume : $12ky^2+8ky-20k$

The expression $12ky^2+8ky-20k$ can also be written as $k\left(12y^2+8y-20\right)$.
$k\left(12y^2+8y-20\right)=k\left(12y^2-12y+20y-20\right)$
$=k[12y(y-1)+20(y-1)]$
$=k(12y+20)(y-1)$
$=4k\times(3y+5)\times(y-1)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $12ky^2+8ky-20k_{\text{is}}\,4k,(3y+5)$ and $(y-1)$.